# Tangram Mathematics 

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# Non-Mathematical Warm-Up <br> The Beauty of the English Language 

## [Personal Selections]

Each phrase in the column on the left below is best used to describe a collections of exactly one of the creatures named in the column on the right. Write the name of that creature in the blank provided. Each name is used only once.
a murmuration of $\qquad$
an army of $\qquad$
a singular of $\qquad$
a charm of $\qquad$
a watch of $\qquad$
a cete of $\qquad$
a herd of $\qquad$
a gam of $\qquad$
a town of $\qquad$
a jubilation of $\qquad$
a muster of $\qquad$
a glitter of $\qquad$
a dule of $\qquad$
a gaggle of $\qquad$
a flock of $\qquad$
a knot of $\qquad$
a pride of $\qquad$
a business of $\qquad$
a mob of $\qquad$
a school of $\qquad$
a labor of $\qquad$
a swarm of $\qquad$
a brace of $\qquad$
alewives
ants
badgers
bees
boars
butterflies
geese
goldfinches
houseflies
kangaroos
kudu
lions
moles
nightingales
peafowl
pheasants
prairie dogs
sheep
skylarks
starlings
toads
turtledoves
whales

## The Beauty of the English Language

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## Tangram Mathematics

The seven-piece tangram puzzle originated in China where it enjoyed a great deal of popularity in the early 1800's. The puzzle involved manipulating the different tangrams into silhouette patterns of people, animals, objects, or geometric figures. Construction of the seven tangram pieces varied from elaborately carved ivory to wooden sets.

Due to the recent emphasis on learning concepts of mathematics through the use of manipulative materials coupled with the natural fascination of puzzles, it is no surprise that the tangram puzzle has become a popular teaching device in the classroom. Many aspects of these shapes lend themselves to the discovery or discussion of concepts such as size, shape, congruence, similarity, properties of polygons, symmetry, and area.

The activities that follow are only some of the examples of the variety of ways in which tangram shapes can be used to reinforce mathematical concepts. Sets of tangram pieces can be made by the students or teachers from colored paper or cardboard. Commercial sets are available and are inexpensive.

## Basic Ideas

- Locate and identify the pieces of a complete set of tangram pieces. You should find the following pieces:

2 small triangles - 1 medium triangle - 1 square -1 parallelogram - 2 large triangles

- Use the tangram pieces to complete the following questions.

How many of the small triangles can fit on the parallelogram? $\qquad$
How many of the small triangles can fit on the square? $\qquad$
How many of the small triangles can fit on a large triangle? $\qquad$
How many medium triangles can fit on a large triangle? $\qquad$
How many ways can the large triangle be covered using the other pieces? $\qquad$
Illustrate some of your solutions to the last question below.


## Picture Shapes

Lots of interesting shapes can be made using tangrams. Some use all of the pieces and others need less or more according to the different shapes. Early exercises may be done with tangram patterns outlining every piece. Following that level, puzzle pictures with an outline of the figure to be covered with no indication of the exact placement could be presented. Increasing in difficulty, puzzles using fewer than the complete set of seven could be used. The most difficult level would be outlines which have been reduced in size and with puzzles requiring more than one set of tangrams.



- Create these shapes using all seven tangram pieces.



## Geometric Shapes

- Take theAdd 2 pieces. Make a $\square$ Trace to record.
- Take a large Add 3 pieces. Make a $\square$ Trace to record.
- Take the . Add 4 pieces. Make a $\square$ Trace to record.


## Area Concepts

- Each of the five tangram shapes below can be covered with the small triangles. How many of the small triangles cover each figure?
$A=1$
$B=$ $\qquad$ $\mathrm{C}=$ $\qquad$
D = $\qquad$
$E=$ $\qquad$

- Which two figures have the same area? Use tangrams to measure.

- Which two figures have the same area? Use tangrams to measure.

- Find the area of each shape below if the tangram square has an area of 7 .



## Similarity

Can you make a triangle similar to the small triangle Tangram piece using 1 Tangram piece? $\qquad$
Verify that the triangles are similar.

If the area of the square Tangram piece is 1 square unit, what is the area of your triangle? $\qquad$
How many different solutions can you find? $\qquad$
Verify that each solution is similar to the small triangle Tangram piece.

What is the area of each different solution? $\qquad$
Draw an illustration of each solution below.

- Can you make a triangle similar to the small triangle Tangram piece using 2 Tangram pieces? $\qquad$
Verify that the triangles are similar.

If the area of the square Tangram piece is 1 square unit, what is the area of your triangle? $\qquad$
How many different solutions can you find? $\qquad$
Verify that each solution is similar to the small triangle Tangram piece.

What is the area of each different solution?
Draw an illustration of each solution below.

- This activity can continue using 3 to 7 Tangram pieces.
- Can you make a square using 1 Tangram piece? $\qquad$
Verify that the squares are similar.

If the area of the square Tangram piece is 1 square unit, what is the area of your square?

How many different solutions can you find? $\qquad$
Verify that each solution is similar to the square Tangram piece.

What is the area of each different solution? $\qquad$
Draw an illustration of each solution below.

- Can you make a square using 2 Tangram pieces? $\qquad$
Verify that the squares are similar.

If the area of the square Tangram piece is 1 square unit, what is the area of your square? $\qquad$
How many different solutions can you find? $\qquad$
Verify that each solution is similar to the square Tangram piece.

What is the area of each different solution? $\qquad$
Draw an illustration of each solution below.

- This activity can continue using 3 to 7 Tangram pieces.
- Additionally, this activity can be done making parallelograms similar to the parallelogram Tangram piece using 1 to 7 Tangram pieces.


## Interior Angles

- Make a shape with the two small Tangram triangles and the parallelogram, like the one shown.

- Determine how many interior angles this figure has.
- Find the measure of each interior angle for this figure.
- What is the sum of the measures of the interior angles?
- Make a triangle using two or more Tangram pieces. How many interior angles does a triangle have? What is the sum of the measures of the interior angles?
- Make a rectangle using two or more Tangram pieces. How many interior angles does a rectangle have? What is the sum of the measures of the interior angles?
- Continue making geometric shapes with differing numbers of sides and complete the chart below.

| Shape | Number of Sides <br> (Interior Angles) | Sum of Measures of <br> Interior Angles |
| :---: | :---: | :---: |
| Triangle | 3 |  |
| Rectangle | 4 |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

- Extension: Is there a way to determine the sum of the interior angles of any polygon without measuring each angle?


## Coordinate Geometry

- Choose an opponent. Using the grid below, place the two large Tangram triangles so that each of the corners align with the intersection of two of the grid lines.
- Taking turns, each person tries to guess the order pairs that mark the corners of the hidden triangles. For each guess, the opponent should respond in one of the four ways below:

Corner Hit - if the corner of one of the triangles touches the ordered pair Side Hit - if the side of triangle touches the ordered pair
Inside Hit - if the triangle covers up the ordered pair
Miss - if the ordered pair is outside the triangle

- The game ends when one player locates the corners of the other player's triangles.



## Classifying Polygons

- Make a list of at least 12 attributes that describe the Tangram pieces or polygons make from 2 Tangram pieces.
- Write each attribute on a separate sticky note. Mix up the notes, face down.
- Randomly select two attribute sticky notes and place them on each of the Venn diagram circles.
- Taking turns (two players) place your Tangram pieces or a 2-piece Tangram polygon into the appropriate section of the Venn diagram. Keep going until there are no different pieces or 2-piece polygons that can be placed.
- Some examples of attributes that may be used on the Attribute sticky notes:

HAS ONE OR MORE RIGHT ANGLES
HAS THREE SIDES
HAS NOT RIGHT ANGLES
IS A QUADRILATERAL
HAS ONE OR MORE OBTUSE ANGLES
ALL SIDES ARE CONGRUENT
NO SIDES ARE CONGRUENT
IS CONVEX
HAS AT LEAST ONE SET OF PARALLEL SIDES
HAS AN AREA LARGER THAN THE MEDIUM TRIANGLE
HAS A REA EQUAL TO THE LARGE TRIANGLE
HAS ONE OR MORE LINES OF SYMMETRY

- The activity may be used with three attribute circles.

Students may play a game in which points are given for the placement of polygons as follows:

Fits none of the attributes $=0$ points
Fits only one attribute $=1$ point
Firs two attributes (in the intersection of two circles) $=2$ points
Fits three attributes = 3 points
Places a polygon in the wrong area $=-1$ point

## Reasoning Skills

- Identify the object that correctly fills the empty space.

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- After solving several puzzles, students can then create their own for classmates to solve. Very often students find it as challenging to create the puzzles at to solve them.
- It is also possible that a puzzle may have more than one correct solution.


## Fractions

If the large Tangram triangle has an area of 1 , identify the area of each of the Tangram pieces.
small triangle $=$ $\qquad$
medium triangle $=$ $\qquad$
square $=$ $\qquad$
parallelogram $=$ $\qquad$
large triangle $=$ $\qquad$

If the parallelogram Tangram piece has an area of $1 / 3$, identify the area of each of the Tangram pieces.
small triangle $=$ $\qquad$
medium triangle $=$ $\qquad$
square $=$ $\qquad$
parallelogram $=$ $\qquad$
large triangle $=$ $\qquad$

If the square Tangram piece has an area of $1 / 4$, identify the area of each of the Tangram pieces.
small triangle $=$ $\qquad$
medium triangle $=$ $\qquad$
square $=$ $\qquad$
parallelogram $=$ $\qquad$
large triangle $=$ $\qquad$

- The seven Tangram pieces can form a square, as shown below.

- If the square above has an area of 1 , find the value of each of the Tangram pieces. small triangle = $\qquad$ medium triangle $=$ $\qquad$
square $=$ $\qquad$
parallelogram = $\qquad$
large triangle = $\qquad$
- Working with a partner, each person make a shape (convex) that has an area of $1 / 2$. Draw an illustration of your solution below.
- Make a different shape than also has an area of $1 / 2$ and draw it below.
- Working with a partner, each person make shapes (convex) that has an area of $1 / 4$. Draw illustrations of your solutions below.
- Working with a partner, each person make shapes (convex) that has an area of $3 / 8$. Draw illustrations of your solutions below.
- Working with a partner, each person make shapes (convex) that has an area of $5 / 8$. Draw illustrations of your solutions below.
- Working with a partner, each person make shapes (convex) that has an area of $3 / 4$.

Draw illustrations of your solutions below.

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